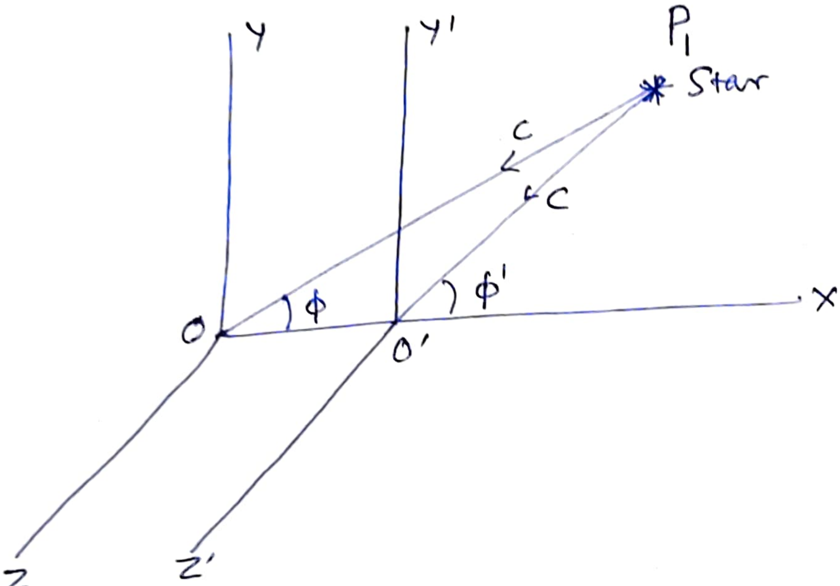
Abberation of light

The phenomenon of abberation was discovered by Bradley in 1927 is very useful to determine the velocity of earth when the velocity of light is known. The phenomenon of abberation

results in the following

"The speed of light is independent of the medium of transmission; but the direction of light rays depends on the motion of the source emitting light relative to the observer."



Imagine the Sun to be a System S while the earth in system S' which is moving with velocity v relative to system S along (+ve) direction Common x-axis. Let the light from a star P be observed by observers O and O' in System S and S' respectively

Let the angle made by a light ray in x-y plane from the Star P at any instant in two systems at O and O' be ϕ and ϕ' respectively.

If any particle P has velocity u in System S and u' in System S', then we have,

$u = u_x \hat{i} + u_y \hat{j} + u_z \hat{k}$

$u' = u_x' \hat{i} + u_y' \hat{j} + u_z' \hat{k}.$

According to Lorentz transformation equations, we have

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\beta^2}}$$

Taking differentials we get

$$dx' = \frac{dx - v\,dt}{\sqrt{1-\beta^2}} \quad \text{——— (1)}$$

$$dy' = dy \quad \text{——— (2)}$$

$$dz' = dz \quad \text{——— (3)}$$

$$dt' = \frac{dt - \frac{v\,dx}{c^2}}{\sqrt{1-\beta^2}} \quad \text{——— (4)}$$

From eqn. (1) & (4), on dividing we get

$$u_x' = \frac{dx'}{dt'} = \frac{dx - v\,dt}{\cancel{\sqrt{1-\beta^2}}} \times \frac{\cancel{\sqrt{1-\beta^2}}}{dt - \frac{v\,dx}{c^2}}$$

$$u_x' = \frac{\frac{dx}{dt} - \frac{v\,\cancel{dt}}{\cancel{dt}}}{\frac{\cancel{dt}}{\cancel{dt}} - \frac{v\,dx}{c^2\,dt}} = \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}}$$

$$u_x' = \frac{u_x - v}{1 - \frac{v}{c^2} u_x} \quad \text{——— (5)}$$

From equation ③ & ④, we get

$$u_y' = \frac{dy'}{dt'} = \frac{dy \sqrt{1-\beta^2}}{dt - \frac{v\,dx}{c^2}} = \frac{\frac{dy}{dt}\sqrt{1-\beta^2}}{\frac{dt}{dt} - \frac{v}{c^2}\frac{dx}{dt}}$$

$$u_y' = \frac{u_y \sqrt{1-\beta^2}}{1 - \frac{v}{c^2} u_x} \quad \text{——— ⑥}$$

The component of velocity c along the direction of x-axis in system S is

$$\left.\begin{aligned} u_x &= c\cos(\pi+\phi) = -c\cos\phi \\ u_y &= c\sin(\pi+\phi) = -c\sin\phi \\ &\text{and in system } S' \text{ is} \\ u_x' &= c\cos(\pi+\phi') = -c\cos\phi' \\ u_y' &= c\sin(\pi+\phi') = -c\sin\phi' \end{aligned}\right\} \text{——— ⑦}$$

From ⑤ & ⑥

$$\frac{u_y'}{u_x'} = \frac{u_y\sqrt{1-\beta^2}}{\cancel{\left(1-\frac{v}{c^2}u_x\right)}} \times \frac{\cancel{\left(1-\frac{v}{c^2}u_x\right)}}{u_x - v}$$

$$\Rightarrow \frac{u_y'}{u_x'} = \frac{u_y\sqrt{(1-\beta^2)}}{u_x - v} \quad \text{——— ⑧}$$

Putting the values from eqn. ⑦, we get

$$\frac{u_y'}{u_x'} = \frac{-c\sin\phi'}{-c\cos\phi'} = \frac{-c\sin\phi\sqrt{(1-\beta^2)}}{-c\cos\phi - v}$$

$$\tan\phi' = \frac{\cancel{c}\sin\phi\sqrt{1-\beta^2}}{\cancel{c}\left(\cos\phi + \frac{v}{c}\right)} = \frac{\sin\phi\sqrt{1-\beta^2}}{\cos\phi + \beta} \quad \text{——— ⑨}$$

$$= \frac{\frac{\sin\phi}{\cos\phi}\sqrt{1-\beta^2}}{\frac{\cos\phi}{\cos\phi} + \frac{\beta}{\cos\phi}} = \frac{\tan\phi\sqrt{1-\beta^2}}{1+\beta\sec\phi}$$

The inverse of equation (9) can be written as

$$\tan\phi = \frac{\sin\phi'\sqrt{1-\beta^2}}{\cos\phi' - \beta} = \frac{\tan\phi'\sqrt{1-\beta^2}}{1-\beta\sec\phi'} \quad \text{--- (10)}$$

From equation (9) & (10) it is obvious that ϕ and ϕ' are not the same in the two system i.e, the direction of light rays depends upon relative motion of the source and observer; therefore the explanation of the phenomenon of abberation is obvious. Any of the eqn. (9) & (10) gives the exact relativistic abberation formula.